MULTIPLE CHOICE: Which of the following statements are contradictions ? **SCORE: _____ / 3 PTS**

[1] $p \oplus \sim p$
[2] $p \leftrightarrow \sim p$
[3] $p \vee \sim p$

(a) none of the above (b) all of the above (c) only [1] (d) only [2]

(e) only [3] (f) only [1] and [2] (g) only [1] and [3] (h) only [2] and [3]

Write the **formal definition** of a valid argument. Use complete sentences and proper English. **SCORE: _____ / 3 PTS**
(Assume that your reader already knows the definition of an argument.)

AN ARGUMENT IS VALID IF AND ONLY IF,
IN ALL CASES WHERE THE PREMISES ARE TRUE,
THE CONCLUSION MUST BE TRUE

Is $p \oplus \sim (q \rightarrow p)$ a tautology ? Show proper justification & state your final answer clearly. **SCORE: ____ / 5 PTS**

p	q	$q \rightarrow p$	$\sim (q \rightarrow p)$	$p \oplus \sim (q \rightarrow p)$	
(1) T	T	T	F	T	
(1) T	F	T	F	T	
(1) F	T	F	T	T	
(1) F	F	T	F	F	←

(1) NOT A TAUTOLOGY

Consider the statement "if $x > 3$, then $\frac{1}{x} \leq 1$". (Assume x is a particular real number.) **SCORE: _____ / 5 PTS**

[a] Write a logically equivalent statement using "is necessary for". **Do not use statement variables.**

$\frac{1}{x} \leq 1$ IS NECESSARY FOR $x > 3$ (1½)

[b] Write the negation of the statement. **Do not use statement variables.**

$x > 3$ AND $\frac{1}{x} > 1$ (2)

[c] Write the inverse of the statement. **Do not use statement variables.**

IF $x \leq 3$, THEN $\frac{1}{x} > 1$ (1½)

Determine if the following argument is valid. **State your final answer clearly.** **SCORE: _____ / 10 PTS**

NOTES: This is *NOT* an essay question. Use the method shown in lecture and section 2.3. Do *NOT* use the Rules of Inference.

If I save a lot of money or I win the lottery, then I can buy an expensive car.
I did not win the lottery.
Therefore, if I cannot buy an expensive car, then I did not save a lot of money.

$m \vee l \rightarrow c$
$\sim l$
$\therefore \sim c \rightarrow \sim m$

	m	l	c	$m \vee l \rightarrow c$	$\sim l$	$\sim c \rightarrow \sim m$	
(1)	T	T	T	T	F	T	
(1)	T	T	F	F	F	F	
(1)	T	F	T	T	T	T	←
(1)	T	F	F	F	T	F	
(1)	F	T	T	T	F	T	
(1)	F	T	F	F	F	T	
(1)	F	F	T	T	T	T	←
(1)	F	F	F	T	T	T	←

(1) FOR IDENTIFYING ALL CRITICAL ROWS

ARGUMENT IS VALID (1)

Prove that the following argument is valid using the Rules of Inference.
Give the reason for each step as shown in lecture.

SCORE: _____ / 9 PTS

$r \rightarrow \sim q$
$p \wedge q$
$(\sim p \vee \sim r) \rightarrow s$
$\therefore s$

① $p \wedge q$	GIVEN
① $\therefore p$	SPEC
① $\therefore q$	SPEC
① $r \rightarrow \sim q$	GIVEN
$\therefore \sim r$	MT (1½)
① $\therefore \sim p \vee \sim r$	GEN
① $(\sim p \vee \sim r) \rightarrow s$	GIVEN
$\therefore s$	MP (1½)